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An Innovative Approach for Solving Fully Triangular Q- Rung Orthopair Fuzzy Linear Fractional Optimization Problem

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ABSTRACT

This paper addresses a nonlinear linear fractional programming (NLFP) problem characterized by parameter uncertainty. To effectively capture imprecision in the model, all coefficients in both the objective function and the constraints are represented using triangular q-rung orthopair fuzzy (q-ROF) numbers (q-ROFNs). The fuzzy formulation is then transformed into an equivalent crisp linear fractional programming model through the application of an appropriate score function. To solve the resulting deterministic model, a novel procedure based on the Fourier–Motzkin elimination technique is developed. The proposed method employs boundary analysis and provides a straightforward and computationally efficient alternative to classical approaches, such as the two-phase simplex method. Finally, a numerical example is included to illustrate the effectiveness and implementation steps of the proposed solution methodology, followed by concluding remarks.

1. Introduction

Linear fractional programming problems (LFPPs) constitute a significant class of optimization models in operations research, particularly suited for real-world situations where performance is better assessed through ratios rather than absolute quantities. Such models arise naturally in applications including cutting-stock and blending problems, transportation and scheduling systems, production planning, and stochastic decision processes.

In many practical environments, decision variables are evaluated in terms of efficiency measures such as cost–benefit ratios, output–input relationships, and productivity indices, where both numerator and denominator are linear functions. When the objective is expressed as a ratio of two linear functions subject to linear constraints, the resulting formulation is known as a linear fractional programming problem. LFPP can be regarded as an extension of classical linear programming, where

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instead of optimizing a single linear objective, the focus shifts to maximizing efficiency. This perspective has also been strongly linked with relative efficiency evaluation frameworks such as data envelopment analysis (DEA), which has been widely applied in performance assessment across sectors including healthcare, education, banking, aviation, and public services [1].

A substantial body of literature has developed solution methodologies for LFPPs, particularly through transformations that convert fractional structures into equivalent linear programming forms. Early foundational ideas in fuzzy decision environments and fractional programming formulations established the theoretical basis for subsequent developments [2–4]. Following this, several algorithmic approaches were introduced, including simplex-based procedures and transformation techniques grounded in classical linear programming theory [5–7]. Alternative solution strategies, such as feasible direction methods, duality-based approaches, interval programming extensions, and decomposition techniques, were later proposed to handle different structural variations of LFPPs [8–13]. With the increasing presence of uncertainty in real-world systems, fuzzy linear fractional programming (FLFP) models have gained considerable attention, where imprecision in parameters is represented using fuzzy numbers and solved through defuzzification or ranking approaches. Further advancements include fully fuzzy, interval-valued, and tri-objective formulations designed to capture lower, middle, and upper bounds of uncertain parameters [14-15].

In more recent developments, LFPPs have been extended into advanced uncertainty modeling frameworks, including neutrosophic and various orthopair fuzzy environments, enabling more flexible representation of indeterminate and inconsistent information [16–22]. In particular, q-ROF sets (q-ROFSs) and their extensions have attracted significant attention due to their higher expressive capacity compared to intuitionistic and Pythagorean fuzzy sets. A wide range of aggregation operators, distance measures, and decision-making models have been developed under this framework to improve handling of uncertainty in complex systems [23–31]. Further contributions include advanced fuzzy aggregation techniques, linguistic extensions, Bonferroni-type operators, improved group decision-making strategies, graph-based models, and tensor-based approaches, all aimed at enhancing flexibility and accuracy in multi-criteria decision environments [32–46].

1.1 Research Gap

Existing literature shows that LFPPs have been widely studied under classical, interval, neutrosophic, and fuzzy environments. However, most approaches depend on crisp transformations or simple-based and aggregation techniques, which are limited in handling higher-order uncertainty. Although neutrosophic and q-ROF frameworks improve uncertainty modeling, their integration into LFPPs remains limited, particularly in unified and computationally efficient forms. Moreover, existing methods are often iterative and computationally intensive, making them less suitable for large-scale or highly uncertain problems. Only a few studies consider transforming q-ROF fractional models into scalar forms using score functions with elimination techniques. In addition, the Fourier–Motzkin elimination method [47] has not been adequately explored for q-ROF LFPPs, indicating a methodological gap in developing boundary-based and efficient solution strategies. Hence, there is a need for an integrated framework combining triangular q-ROF modeling with efficient transformation and elimination techniques to reduce computational complexity while ensuring solution accuracy.

1.2 Contributions to the Paper

The main contributions of this study are summarized as follows:

i. This study investigates a linear fractional programming problem (FPP) under uncertainty, formulated as a nonlinear FPP (NLFPP).
ii. All parameters in the objective function and constraints are modeled using triangular q-ROFNs to effectively handle uncertainty and imprecision.
iii. A score function is employed to convert the fuzzy optimization model into an equivalent scalar linear FPP.
iv. A novel solution approach based on the Fourier–Motzkin elimination technique [47] is developed to obtain the optimal solution of the transformed model.
v. The proposed framework is grounded in boundary-based analysis, making it systematic, transparent, and easy to implement.
vi. Compared with classical approaches such as the two-phase simplex method, the proposed method is computationally more efficient and reduces computational burden.

1.3 Structure of the paper

This section is organized as follows. Section 2 presents the preliminary concepts and foundational definitions required for the development of the proposed framework. Section 3 formulates the linear fractional programming (LFP) problem under triangular q-ROFNs, establishing the mathematical model under uncertainty. Section 4 develops the proposed solution procedure for solving the formulated model in a systematic manner. Section 5 provides a numerical example to illustrate and clarify the step-by-step implementation of the solution approach. Section 6 evaluates the performance of the proposed algorithm by discussing its strengths and limitations. Section 7 presents a detailed discussion of the results obtained and their implications. Finally, Section 8 concludes the paper and highlights future research directions.

2. Preliminaries

This section outlines the key concepts and fundamental notions associated with q-ROFNs. It also describes the basic operational rules and introduces the ranking mechanism employed for their evaluation and comparison.

Definition 1. Consider H be a finite set and let ζ be an arbitrary component of H. A fuzzy set (FS) [48], $\tilde{A}$ on H is characterized by the collection

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in H\},$$

here $\mu_{\tilde{A}}(\zeta): H \to [0,1]$. This value represents the degree to which x belongs to the fuzzy set $\tilde{A}$, and $\mu_{\tilde{A}}$ is known as the MD.

Definition 2. Let Rrepresent the set of real numbers. A fuzzy set $\tilde{A}$defined on R, characterized by a membership function (MF) $\mu_{\tilde{A}}: R \to [0,1]$, is termed as a fuzzy number (FN) , [48], provided that the following properties are satisfied:

Normality: There exists at least one point $x^0 \in R$for which $\mu_{\tilde{A}}(x^0) = 1$.

Convexity: For any $x, y \in R$and any $\gamma \in [0,1]$, the MF obeys $\mu_{\tilde{A}}(\gamma x + (1-\gamma)y) \geq \min(\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y))$.

Piecewise Continuity: The function $\mu_{\tilde{A}}(x)$is piecewise continuous across the domain$R$.

Definition 3. Consider a universe of discourse H. A q-ROF set [4], symbolized by Δ, is defined as $\Delta = \{\langle x, \mu_\Delta(x), \nu_\Delta(x)\rangle \mid x \in H\}$, where the mappings $\mu_\Delta: H \to [0,1]$and $\nu_\Delta: H \to [0,1]$denote the MD and non-MDs (NMD) of the element$x$, respectively. These degrees must satisfy the restriction $0 \leq (\mu_\Delta(x))^q + (\nu_\Delta(x))^q \leq 1, q \geq 1,$ the hesitancy degree corresponding to $x$is expressed as$\chi_\Delta = (1 - (\mu_\Delta)^q - (\nu_\Delta)^q))^{1/q}$.

The membership space q-ROF set for different values of q is presented in Figure 1.

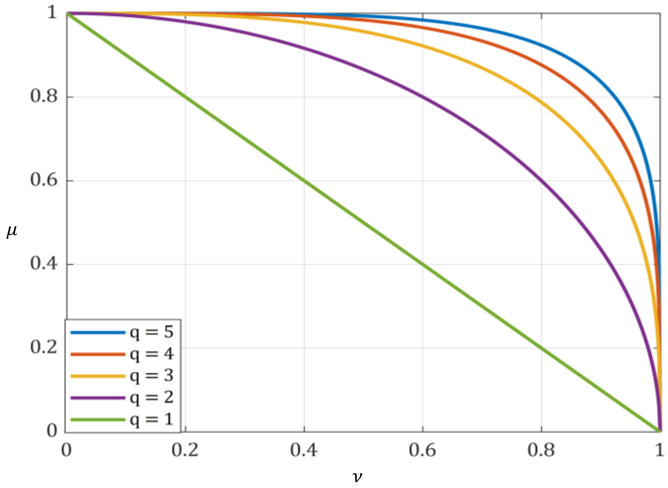

Fig. 1. Diagrammatic representation of q- ROFN

Definition 4. A triangular q-ROFN (TQROPFN) [35]can be expressed as

$$\widetilde{\Theta}^{QRF} = \langle (a_1, a_2, a_3); \varpi, \xi \rangle$$

Where the real-valued parameters satisfy $a_1 \leq a_2 \leq a_3$. The MF and non-MF are described through piecewise functions defined over the real line:

$$\varpi_{\widetilde{\Theta}^{QRF}}(\mathrm{x}) = \begin{cases} \left(\dfrac{x - a_1}{a_2 - a_1}\right)\varpi, & x \in [a_1, a_2), \\ \varpi, & x = a_2, \\ \left(\dfrac{a_3 - x}{a_3 - a_2}\right)\varpi, & x \in [a_2, a_3), \\ 0, & \text{otherwise}, \end{cases}$$

$$\xi_{\widetilde{\Theta}^{QRF}}(\mathrm{x}) = \begin{cases} \left(\frac{a_3 - x}{a_3 - 2}\right)\xi, & x \in [a_1, a_2), \\ \xi, & x = a_2, \\ \left(\frac{a_3 - x}{a_3 - 2}\right)\xi, & x \in [a_2, a_3), \\ 1, & \text{otherwise}, \end{cases}$$

Where μ_a and ν_a represent the highest admissible MD and NMDs, respectively. These parameters belong to the interval [0,1] and must satisfy the orthopair condition

$$\left(\varpi_{\widetilde{\Theta}^{QRF}}\right)^q + \left(\xi_{\widetilde{\Theta}^{QRF}}\right)^q \leq 1$$

The quantities $\varpi$and ξ quantify the degrees of support and opposition inherent in the fuzzy representation ($\varpi^q + \xi^q \leq 1$).

Definition 5. Consider two triangular q-ROFNs (TQROPFNs)

$$\tilde{\Delta}^{QRF} = \langle (\gamma_1, \gamma_2, \gamma_3); \mu_{\tilde{\Delta}}, \nu_{\tilde{\Delta}} \rangle \text{and } \widetilde{\nabla}^{QRF} = \langle (\lambda_1, \lambda_2, \lambda_3); \mu_{\widetilde{\nabla}}, \nu_{\widetilde{\nabla}} \rangle$$

The fundamental operational laws for these FNs [35] are defined as follows:

i. Addition

$$\tilde{\Delta}^{QRF} \oplus \widetilde{\nabla}^{QRF} = \left\langle (\gamma_1 + \lambda_1, \gamma_2 + \lambda_2, \gamma_3 + \lambda_3); \left(\mu_{\tilde{\Delta}}^q + \mu_{\widetilde{\nabla}}^q - \mu_{\tilde{\Delta}}^q \mu_{\widetilde{\nabla}}^q\right)^{\frac{1}{q}}, \nu_{\tilde{\Delta}} \nu_{\widetilde{\nabla}} \right\rangle$$

ii. Multiplication

$$\tilde{\Delta}^{QRF} \otimes \tilde{\nabla}^{QRF} = \left\langle (\gamma_1\lambda_1, \gamma_2\lambda_2, \gamma_3\lambda_3); \mu_{\tilde{\Delta}}\mu_{\tilde{\nabla}}, \left(\nu_{\tilde{\Delta}}^q + \nu_{\tilde{\nabla}}^q - \nu_{\tilde{\Delta}}^q \nu_{\tilde{\nabla}}^q\right)^{\frac{1}{q}} \right\rangle$$

iii. Division

$$\frac{\tilde{\Delta}^{QRF}}{\tilde{\nabla}^{QRF}} = \left\langle \left(\frac{\gamma_1}{\lambda_3}, \frac{\gamma_2}{\lambda_2}, \frac{\gamma_3}{\lambda_1}\right); \mu_{\Delta}\mu_{\nabla}, \left(\nu_{\tilde{\Delta}}^q + \nu_{\tilde{\nabla}}^q - \nu_{\tilde{\Delta}}^q \nu_{\tilde{\nabla}}^q\right)^{\frac{1}{q}} \right\rangle$$

iv. Scalar Multiplication

$$\delta\tilde{\Delta}^{QRF} = \left\langle (\delta\gamma_1, \delta\gamma_2, \delta\gamma_3); \left(1 - (1 - \mu_{\Delta}^q)^{\delta}\right)^{\frac{1}{q}}, \nu_{\tilde{\nabla}}^{\delta} \right\rangle, \delta > 0$$

v. Power Operation

$$\left(\tilde{\Delta}^{QRF}\right)^{\delta} = \langle (\gamma_1^{\delta}, \gamma_2^{\delta}, \gamma_3^{\delta}); \mu_{\Delta}^{k}, \left(1 - (1 - \nu_{\Delta}^q)^{\delta}\right)^{1/q} \rangle, \delta > 0$$

Definition 6. Let $\tilde{\Delta}^{QRF} = \langle (\gamma_1, \gamma_2, \gamma_3); \mu_{\tilde{\nabla}}, \nu_{\tilde{\nabla}} \rangle$ denote a triangular q-ROFN (TQROPFN). The score function [35], which is used to assess and compare such FNs, is defined as

$$SC(\tilde{\Delta}^{QRF}) = \frac{(\gamma_1 + \gamma_2 + \gamma_3)}{8}\left(1 + \mu_{\tilde{\Delta}}^q - \nu_{\tilde{\Delta}}^q\right)$$

Definition 7. Consider two TQROPFNs $\tilde{\Delta}^{QRF} = \langle (\gamma_1, \gamma_2, \gamma_3); \mu_{\Delta}, \nu_{\Delta} \rangle$ and $\nabla^{QRF} = \langle (\lambda_1, \lambda_2, \lambda_3); \mu_{\nabla}, \nu_{\nabla} \rangle$. The comparative relation between these q-ROFNs [35] is established through their score values. Specifically, Δ^{QRF} is regarded as superior (or inferior) to ∇^{QRF} if $SC(\Delta^{QRF}) > SC(\nabla^{QRF})$(or $SC(\Delta^{QRF}) < SC(\nabla^{QRF})$).

The two FNs are considered equivalent whenever their score functions coincide, that is, $SC(\Delta^{QRF}) = SC(\nabla^{QRF})$.

3. Problem Statement and solution concepts

A general form of LFPP can be formulated as follows:

$$max\, F(x) = \frac{P(x)}{Q(x)} = \frac{c^t x + \varsigma}{d^t x + \varrho}$$

Subject to

$$x \in \Delta = \{x \in \mathbb{R}^n : h(x) = Ax - b \le 0, x \ge 0\}.$$

Where, $c, d \in \mathbb{R}^n$; $b \in \mathbb{R}^m$; $A \in \mathbb{R}^{m\times n}$, and $\varsigma, \varrho \in \mathbb{R}$.

A linear FPP with fully triangular Q- rung orthopair FN can be formulated as

$$\text{(P1)}\ \ max\, \tilde{Z}^{QRF} = \frac{\left(\tilde{c}^{QRF}\right)^T x_j \oplus \tilde{\varsigma}^{QRF}}{\left(\tilde{d}^{QRF}\right)^T x_j \oplus \tilde{\varrho}^{QRF}}, j = 1, 2, \dots, n$$

Subject to

$$\tilde{X}^{QRF} = \left\{\sum_{j=1}^{n} \tilde{a}_{ij}^{QRF} x_j (\le, =, \ge) \tilde{b}_i^{QRF}, i = 1,2, \dots, m; x_j \ge 0, j = 1, 2, \dots, n\right\}$$

Assume that $\tilde{d}^{QRF} x_j \oplus \tilde{\varrho}^{QRF} \succ 0; \forall j$, $\emptyset = \tilde{X}^{QRF}$ represents the feasible domain.

Definition 8. The $\overline{x_j}$ which satisfies the condition in problem P1 is called a triangular Q- rung orthopair fuzzy optimization solution. Furthermore, if $\overline{x_j}$ is a triangular Q- rung orthopair fuzzy vector on its feasible domain, then it is said to be triangular Q- rung orthopair fuzzy optimization solution of problem P1.

Based on the score function definition, P1 is converted into

$$\text{(P2)}\ \ \max Z = \frac{c_j^T x_j + \varsigma}{d_j^T x_j + \varrho}, j = 1, 2, \dots, n$$

Subject to

$$X = \left\{\sum_{j=1}^{n} a_{ij} x_j (\le, =, \ge) b_i, i = 1,2, \dots, m; x_j \ge 0, j = 1, 2, \dots, n\right\}$$

A triangular Q- rung orthopair FNs are used to represent uncertainty and indeterminacy in this problem. They extend the traditional real numbers to handle vague and imprecise information, making NLFP suitable for real- world decision- making scenarios where precise data may be lacking.

4. Solution procedure

In this section, solution technique to obtain triangular Q- rung orthopair fuzzy solution is introduces which is based on the Fourier- Motzkin Elimination Method [47].

Step1: Formulate P1.

Step 2: Apply the score function definition to obtain P2.

Step 3: Solve P2 using the Fourier- Motzkin Elimination Method [47].

Rewrite P2 as follows:

(P3) $\max \mathrm{Z} = \frac{\mathrm{c}_{\mathrm{j}}^{\mathrm{T}}\mathrm{x}_{\mathrm{j}}+\varsigma}{\mathrm{d}_{\mathrm{j}}^{\mathrm{T}}\mathrm{x}_{\mathrm{j}}+\varrho}, \mathrm{j} = 1, 2, \dots, \mathrm{n}.$

Subject to

$$\left(\mathrm{c}_j^T x_j + \varsigma\right)Z - \left(\mathrm{d}_j^T \mathrm{x}_{\mathrm{j}} + \varrho\right)Z \geq 0, j = 1,2, \dots, n,$$
$$-\left(\mathrm{c}_j^T x_j + \varsigma\right)Z + \left(\mathrm{d}_j^T \mathrm{x}_{\mathrm{j}} + \varrho\right)Z \geq 0, j = 1,2, \dots, n,$$
$$-\sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{a}_{ij}\mathrm{x}_{\mathrm{j}} \geq -\mathrm{b}_i, \mathrm{i} = 1,2, \dots, \mathrm{m},$$
$$\mathrm{x}_{\mathrm{j}} \geq 0, \mathrm{j} = 1, 2, \dots, \mathrm{n}.$$

After having three classes of $\mathrm{x}_{\mathrm{j}} \geq 0, \mathrm{j} = 1, 2, \dots, \mathrm{n}$, combine the inequalities and reduce them one by one that's by iteration. A triangular Q- rung orthopair fuzzy are used to represent uncertainty and indeterminacy in this problem. They extend the traditional real numbers to handle vague and imprecise information, making LFP suitable for real- world decision- making scenarios where precise data may be lacking.

Remark 1. In the case of $0 \geq d$ and d is positive, then we have infeasible solution otherwise the solution is feasible.

5. Numerical Example

Consider the following problem

$$\max \tilde{Z}^{QRF} = \frac{[(7,9,11); (0.5,0.3)^q]x_1 + [(5,6,7); (0.3,0.2)^q]x_2}{[(7,9,11); (0.5,0.3)^q]x_1 + [(3,4,5); (0.3,0.1)^q]x_2 + [(1,2,3); (0.3,0.1)^q]}$$

Subject to (4)

$$[(5,6,7); (0.3,0.2)^q]x_1 + [(7,9,11); (0.5,0.3)^q]x_2 \preccurlyeq [(18,20,22); (0.8,0.1)^q],$$
$$[(7,9,11); (0.5,0.3)^q]x_1 + [(3,4,5); (0.3,0.1)^q]x_2 \preccurlyeq [(17,18,19); (0.5,0.1)^q],$$
$$x_1, x_2 \geq 0.$$

Step 2: The above problem with $q = 3$, becomes

$$\max Z = \frac{4.941x_1+3.057x_2}{4.941x_1+2.052x_2+1.026}$$

Subject to

$$2.052x_1 + 4.941x_2 \leq 15.11,$$
$$4.941x_1 + 2.052x_2 \leq 10.116, \qquad (1)$$
$$x_1, x_2 \geq 0.$$

Step 3: Problem (1) becomes

$\max Z$

Subject to

$$(4.941x_1 + 2.052x_2 + 1.026)Z + (4.941x_1 + 3.057x_2)Z + Z \geq 0,$$
$$-(4.941x_1 + 2.052x_2 + 1.026)Z - (4.941x_1 + 3.057x_2)Z - Z \geq 0,$$
$$-3.057x_1 - 4.941x_2 \geq -15.11,$$

$-4.941x_1 - 2.052x_2 \geq -10.116,\ x_1, x_2 \geq 0.$

The previous problem can be rewritten as

$$\max Z$$

$$\text{Subject to}\quad 4.941(Z-1)x_1 + (2.052Z - 3.057)x_2 + Z \geq 0,$$
$$-4.941(Z-1)x_1 - (2.052Z - 3.057)x_2 - Z \geq 0,$$
$$-3.057x_1 - 4.941x_2 \geq -15.11, \tag{2}$$
$$-4.941x_1 - 2.052x_2 \geq -10.116,$$
$$x_1, x_2 \geq 0.$$

Now, assume that the bound of $1 \leq Z < \frac{3.057}{2.052}$, then equatins of (2) become

$$-x_1 + \frac{2.052Z-3.057}{4.941(Z-1)}x_2 - \frac{Z}{4.941(Z-1)} \geq 0,$$
$$x_1 + \frac{2.052Z-3.057}{4.941(Z-1)}x_2 + \frac{Z}{4.941(Z-1)} \geq 0,$$
$$-x_1 - \frac{4.941}{3.057}x_2 \geq -4.941, \tag{3}$$
$$-x_1 - \frac{2.052}{4.941}x_2 \geq -2.052,$$
$$x_1 \geq 0,$$
$$x_2 \geq 0.$$

Eliminate x_1 from the system (3) to obtain

$$-x_2 + \frac{3.057Z}{18.1405Z-15.0685} \geq -\frac{74.634(Z-1)}{18.1405Z-15.0685},$$
$$-x_2 + Z \geq -10(Z-1),$$
$$x_2 - \frac{Z}{2.052Z - 3.057} \geq 0,$$
$$-x_2 \geq -3.052, \tag{4}$$
$$-x_2 \geq -4.941,$$
$$x_2 \geq 0.$$

Equations (4) after eliminating x_2 become

$$Z \geq \frac{74.634}{77.691}, Z \geq \frac{10}{11},$$
$$Z \leq \frac{9.345}{7.273},$$
$$Z \leq \frac{15.105}{10.139},$$
$$-177Z^2 + 440Z - 228 \geq 0,$$
$$22.572Z^2 - 47.042Z + 30.57 \geq 0,$$

It's obvious that the optimum value is $Z = \frac{9.345}{7.273}$, at $x_1 = 0$ and $x_2 = 3$.

By utilizing q- rung arithmetic operations, the q- rung orthopair fuzzy optimal solution is $\tilde{Z}^{\mathrm{QRF}} = \left[\left(\frac{(15,17,19)}{(9,12,14)}\right); (0.4.\,0.1)^q\right] = \left[\left(\frac{15}{14}, \frac{17}{12}, \frac{19}{9}\right);\ (0.4, 0.1)\right]$ at $x_1 = [(0,0,0); (0.1, 0.1)^q]$ and $x_2 = [(5,6,7); (0.3, 0.2)^q]$.

6. Evaluation of the Proposed Algorithm's Strengths and Limitations

A primary strength of the proposed approach lies in its innovative integration triangular Q- rung orthopair fuzzy analysis, linear fractional optimization, Fourier-Motzkin elimination method, and decision-maker expertise. The proposed method integrates a parametric study for efficient search space exploration with a linear fractional analysis that ranks alternatives based on the decision maker's (DM) preferences, explicitly including the DM's judgment in the evaluation. Nevertheless, implementing the proposed algorithm in real-world scenarios may involve certain limitations, including:

The method is limited in that it cannot account for all possible scenarios within the infinite parametric space. However, it is important to recognize that no existing approach can effectively address problems involving an infinite scenario domain.

It is not possible to establish a single standardized technique for identifying the scenarios of interest to the decision maker (DM), as the DM's perspectives and weighting preferences differ across individuals.

Several factors must be taken into account, including: (i) The feasibility of modeling the problem via a Fourier- Motzkin elimination method is being assessed, and (ii) The ability to address the chosen scenarios using a Fourier- Motzkin elimination framework and determine their precise optimal solutions.

7. Discussion of the results

For the q- rung orthopair fuzzy optimum value $\left[\left(\frac{15}{14},\frac{17}{12},\frac{19}{9}\right);\ (0.4, 0.1)\right]$,the total maximum profit will be $\frac{15}{14}, < Z < \frac{17}{12}$and the total maximum profit $\frac{15}{14} < Z < \frac{19}{9}$, the overall levelof acceptance or satifcation or the truthfulness is 4%. Also, for the remaining value of the total profit, The MF is $\vartheta_{\tilde{a}^{\mathrm{TrSFn}}} \times 100$ where x denotes the total profit and $\psi_{\tilde{a}}($x is given as

$$\psi_{\tilde{a}}(x) = \begin{cases} \varpi_{\tilde{a}}\left(\frac{x-\frac{15}{14}}{\frac{17}{12}-\frac{15}{14}}\right), \text{for } \frac{15}{14} \leq x \prec \frac{17}{12}, \\ \varpi_{\tilde{a}}, \qquad x = \frac{17}{12}, \\ \varpi_{\tilde{a}}\left(\frac{\frac{19}{9}-x}{\frac{19}{9}-\frac{17}{12}}\right), \text{for} \frac{17}{12}, \leq x \prec \frac{19}{9} \\ 0,\ \text{for } x \prec \frac{15}{14} \text{ or } x \succ \frac{19}{9} \end{cases}$$

$$\varphi_{\tilde{a}}(x) = \begin{cases} \vartheta_{\tilde{a}}\left(\frac{x-\frac{15}{14}}{\frac{17}{12}-\frac{15}{14}}\right), \text{for } \frac{15}{14} \leq x \prec \frac{17}{12} \\ \vartheta_{\tilde{a}}, \qquad x = \frac{17}{12} \\ \vartheta_{\tilde{a}}\left(\frac{\frac{19}{9}-x}{\frac{19}{9}-\frac{17}{12}}\right), \text{for } \frac{17}{12}, \leq x \prec \frac{19}{9} \\ 0,\ \text{for } x \prec \frac{15}{14} \text{ or } x \succ \frac{19}{9} \end{cases}$$

Thus, it is concluded that the total neutrosophic profit lies within the interval $15/14$ to $19/9$, along with the corresponding MF, MF, and indeterminacy degrees. Based on these results, the decision-maker can appropriately plan the profit and production constraints.

8. Conclusion and Future works

A linear FPP in which all parameters are represented by q-ROFNs is investigated. A solution approach based on the Fourier–Motzkin elimination technique is proposed to obtain the optimal solution under q-ROF information. The method is computationally efficient, simple to implement, and requires less processing time compared with conventional approaches. Furthermore, the obtained results can be validated through graphical analysis, the simplex method, and other existing techniques. Future research may extend this framework to nonlinear and multi-objective fractional programming models within q-ROF environments. The impact of alternative ranking and score functions on solution stability and accuracy also warrants further investigation. In addition, the boundary-based solution procedure can be enhanced and benchmarked against advanced optimization methods to improve performance in large-scale problems. The incorporation of other uncertainty representations, such as interval-valued and hesitant q-ROFSs, would further generalize the model. Finally, applying the proposed approach to real-world problems in engineering, finance, and supply chain management would help demonstrate its practical effectiveness and robustness.

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Conflicts of Interest
The authors declare no conflicts of interest.

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