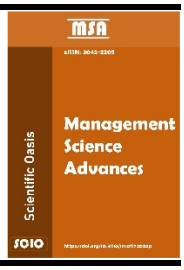
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A Comprehensive Review of Fractional Inventory Problems: Models, Applications, and Future Directions

Rakibul Haque[1], Magfura Pervin[2], Kamal Hossain Gazi[1], Sankar Prasad Mondal[1,*]

[1] Department of Applied Mathematics, Maulana Abul Kalam Azad University of Technology, West Bengal, India
[2] Department of Mathematics, Bangabasi Evening College, West Bengal, India

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ABSTRACT

The fractional inventory problem extends classical inventory models by incorporating fractional calculus to better capture memory effects and hereditary properties in real-world systems. This paper reviews the development and applications of fractional-order inventory models, highlighting their advantages over traditional models in addressing non-instantaneous dynamics and demand variability. Key approaches such as fractional-order differential equations, optimization under constraints, fuzzy logic integration, and hybrid demand functions are discussed. Additionally, the paper outlines current trends in research, identifies challenges, and suggests future directions for model improvement and practical deployment in supply chain systems. Recent studies have further expanded the applicability of fractional inventory models by integrating stochastic demand patterns, sustainability considerations, and multi-echelon supply chain structures. Researchers have demonstrated that fractional-order models provide improved flexibility and forecasting accuracy compared to integer-order formulations, particularly in environments characterized by uncertainty and long-term dependency effects. Moreover, advancements in computational techniques and numerical methods have enabled the practical implementation of these models in complex industrial scenarios. The growing intersection between fractional calculus, artificial intelligence, and data-driven optimization is expected to further enhance inventory decision-making and contribute to the development of resilient and adaptive supply chain systems.

1. Introduction

Fractional inventory problems integrate classical inventory theory with fractional calculus to better capture memory, aftereffects, and long-range dependencies in demand, deterioration, and replenishment processes. Early foundations in inventory modeling emerged with the deterministic Economic Order Quantity (EOQ) principles of Harris [1], which were further generalized in optimal

* *Corresponding author.*
E-mail address: sankar.mondal02@gmail.com

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inventory policy studies by Arrow *et al.* [2], dynamic lot sizing by Wagner & Whitin [3], and the broader analytical frameworks of Hadley & Whitin [4]. Initial models largely assumed instantaneous system responses without memory. However, empirical evidence of perishability, lead-time variability, and backlog patterns led to incorporating more realistic features such as exponential decay [5], Weibull-distributed deterioration rate [6], permissible delay in payments [7], and time-dependent backlogging [8]. Reviews by Nahmias [9] and developments by Silver *et al.* [10] further emphasized dynamic and stochastic considerations in inventory systems.

Alongside this, Caputo [11] introduces fractional calculus in applied form. Samko *et al.* [12] and Podlubny [13] provided tools to model nonlocal, history-dependent dynamics. Hilfer [14] contributed to the physical interpretation of fractional calculus by linking fractional dynamics with anomalous diffusion and complex media behavior, which expanded its applicability beyond pure mathematics. Kilbas *et al.* [15] advanced the theoretical foundation of fractional differential equations by providing rigorous definitions, solution techniques, and applications, thereby establishing a solid mathematical framework for nonlocal operators in applied problems.

Magin [16] introduced fractional calculus in bioengineering contexts, demonstrating its ability to represent complex biological tissue responses more accurately than classical integer-order models. Sabatier *et al.* [17] compiled advances in fractional calculus, particularly emphasizing its role in real-world systems across physics and engineering, and encouraged cross-disciplinary adoption. Monje *et al.* [18] extended fractional calculus into control engineering by developing practical methods for modeling and controlling fractional-order systems, thus bridging theoretical results and engineering applications. Diethelm [19] focused on numerical methods for solving fractional differential equations, providing reliable computational schemes essential for practical applications in science and engineering. Trigeassou *et al.* [20] investigated the transient and steady-state behavior of fractional-order systems, offering state-variable approaches that improved understanding of system memory effects.

Classical inventory management models are generally based on exact data and clearly defined parameters. However, critical factors related to inventory control policy, such as demand patterns, lead times, and holding costs, are often uncertain or imprecise. To address this issue, fuzzy logic [21] serves as a powerful framework for managing vagueness and ambiguity in such data. Since then, researchers have increasingly extended inventory models using fuzzy concepts, making more realistic and flexible solutions under uncertain conditions where precise information is unavailable. Park [22] was among the first to apply fuzzy set theory in EOQ models, demonstrating how uncertain demand could be modelled without relying on probability assumptions. Therefore, the application of fractional calculus in combination with fuzzy logic to inventory control policies has emerged as a more realistic approach. By incorporating fractional derivatives and integration, and fuzzy logic, researchers have been able to design more robust models. For example, Pakhira *et al.* [23] developed an inventory model with demand varying linearly over time by applying these concepts. In their study, fuzzy parameters were handled through defuzzification techniques such as the signed distance method and the graded mean integration method. Rahaman *et al.* [24] introduced an inventory framework that integrates fractional calculus with fuzzy logic, formulating the problem using fuzzy fractional differential equations. This approach provided a more flexible representation of uncertainty while also reflecting the system's memory-dependent characteristics. These studies highlight how fuzzy theory and fractional calculus can significantly advance inventory modelling by bridging the gap between mathematical rigour and practical applicability.

1.1 Motivation of this Study

The motivation of this study is briefly discussed in this section. There are some limitations in classical calculus, especially fractional calculus, to be considered in the context of the "memory" and hereditary traits inherent in inventory supply chain management. Analyzing several fractional-order inventory models, researchers can structure a more accurate inventory model and determine how previous and current demand influence stock decisions. These inventory models are specifically effective for perishable items, emergency medicines, and fluctuating markets, where conventional linear models are insufficient. The fundamental aims of this study are to determine the gaps between several complex mathematical models and practical inventory problems and provide a structural framework that improves cost optimization and systems stability by treating time as a continuous, memory-dependent variable rather than a series of isolated scenarios.

1.2 Review Methodology

The methodology of this review article is clearly described in this section. This study analyzes the existing literature of fractional inventory models from a systematic and structured viewpoint. All the related research articles and books are collected from significant scholarly databases, including Scopus, Google Scholar, ResearchGate, and Web of Science, using several related keywords. The studies are screened and selected based on importance, relevance, quality of publication, and contribution to the fractional-order mathematical model in inventory management systems. The selected articles and books are classified according to techniques, models, solution approaches, and applications in real-world fields. Advantages of fractional models, strengths, challenges, and limitations of the studies are discussed as a comparative analysis. Lastly, the emerging trends and research gaps of this section are identified to propose future research directions.

This section describes the structure of this paper in detail. The introduction section is discussed in Section 1. The brief theoretical background is covered in Section 2. The classification of models is presented in Section 3. Comparative analysis is discussed in Section 4. Furthermore, Section 5 covers the methodologies for fractional inventory models. Section 6 presents the applications of the fractional inventory models, followed by Section 7, which covers the challenges and limitations of this review model. Future directions of this study are presented in Section 8. Finally, the conclusion of the review study is covered in Section 9.

2. Brief Theoretical Background

Fractional calculus allows differentiation and integration of arbitrary (non-integer) order. In this section, the definitions of fractional derivative and integral in the Caputo and Riemann-Liouville sense are presented. The memory-dependent behavior of fractional-order systems is also discussed briefly at the end.

2.1 Caputo Derivative

Definition 1. Suppose $\hbar(x)$ is a real-valued continuous function defined on $[a, b]$ and $\hbar(x)$ has a bounded derivative in $[a, b]$. Then the Left Caputo fractional derivative of the function $\hbar(x)$ of fractional order α is as follows:

$$ {}_{a}^{C}D_{x}^{\alpha}\big(\hbar(x)\big) = \frac{1}{\Gamma(m-\alpha)}\int_{a}^{x}(x-\zeta)^{(m-\alpha-1)}\,\hbar^{m}(\zeta)d\zeta \tag{1}$$

where m is an integer satisfying $0 \leq m-1 < \alpha < m$.

The Right Caputo fractional derivative of the function $\hbar(x)$ of fractional order α is denoted and presented as follows:

$$ {}_{x}^{C}D_{b}^{\alpha}\left(\hbar(x)\right) = \frac{1}{\Gamma(m-\alpha)}\int_{x}^{b}(x-\zeta)^{(m-\alpha-1)}\,\hbar^{m}(\zeta)d\zeta \tag{2}$$

where $0 \le m-1 < \alpha < m$.

2.2 Riemann-Liouville Integration

Definition 2. [15] The left-sided Riemann-Liouville fractional integral ${}^{RL}\mathcal{J}_{a}^{\beta}\hbar(x)$ of order $\beta > 0$ of a continuous function $\hbar$: $[a,b] \to \mathbb{R}$, which has a bounded derivative in $[a,b]$ is defined as:

$$ {}^{RL}\mathcal{J}_{a}^{\beta}\hbar(x) = \frac{1}{\Gamma(\beta)}\int_{a}^{x}\frac{\hbar(s)}{(x-s)^{1-\beta}}ds \tag{3}$$

for all $x \in [a,b]$. The right-sided Riemann-Liouville fractional integral ${}^{RL}\mathcal{J}_{b-}^{\beta}\hbar(x)$ of order β of $\hbar(x)$ is the similitude of the left-sided one, where integration is taken from the point x up to the upper bound b instead of from a lower bound a to x. So:

$$ {}^{RL}\mathcal{J}_{b-}^{\beta}\hbar(x) = \frac{1}{\Gamma(\beta)}\int_{x}^{b}\frac{\hbar(s)}{(x-s)^{1-\beta}}ds \tag{4}$$

Pakhira *et al.* [23] studied the memory effects of an inventory model using Riemann-Liouville integration.

2.3 Riemann-Liouville Derivative

Definition 3. [15] The left-sided R-L fractional derivative ${}^{RL}\mathcal{D}_{a}^{\alpha}\hbar(x)$ of order α of a continuous function is defined as:

$$ {}^{RL}\mathcal{D}_{a}^{\alpha}\hbar(x) = \frac{d^{m}}{dx^{m}}\left({}^{RL}\mathcal{J}_{a}^{m-\alpha}\hbar(x)\right) = \frac{1}{\Gamma(\mathrm{m}-\alpha)}\left(\frac{d}{dx}\right)^{m}\int_{a}^{x}\frac{\hbar(s)}{(x-s)^{\alpha+1-m}}ds \tag{5}$$

for all $x \in [a,b]$ and $0 \le m-1 < \alpha < m$.

The right-sided RL fractional derivative ${}^{RL}\mathcal{D}_{b-}^{\alpha}\hbar(x)$ of order α is the similitude of the left-sided derivative. Here, the integration is taken from the evaluation point x up to the right endpoint b of the domain. So:

$$ {}^{RL}\mathcal{D}_{b-}^{\alpha}\hbar(x) = (-1)^{m}\frac{d^{m}}{dx^{m}}\left({}^{RL}\mathcal{J}_{b-}^{m-\alpha}\hbar(x)\right) = \frac{(-1)^{m}}{\Gamma(\mathrm{m}-\alpha)}\left(\frac{d}{dx}\right)^{m}\int_{x}^{b}\frac{\hbar(s)}{(x-s)^{\alpha+1-m}}ds \tag{6}$$

Pakhira *et al.* [23] studied the memory effects of an inventory model using the Riemann-Liouville derivative.

2.4 Memory-Dependent Derivatives

The derivative of a function $\hbar(t)$ through the kernel function $k(t-\zeta)$ can be presented as:

$$ D[\hbar(t)] = \int_{a}^{t}k(t-\zeta)\hbar'(\zeta)d\zeta \tag{7}$$

The definition of the derivative is kernel-dependent and thus alters with the choice of an appropriate kernel. Specifically:

i. When the kernel $k(t-\zeta)=\delta(t-\zeta)$, where $\delta(.)$ denotes the standard Dirac delta function, then Equation (7) reduces to $D[\hbar(t)]=\hbar'(t)$, which corresponds to the classical integer-order derivative.

ii. When the kernel $k(t-\zeta)=\frac{(t-\zeta)^{\alpha-2}}{\Gamma(\alpha-1)}$, then Equation (7) reduces to:

$$D[\hbar(t)]=\int_a^t \frac{(t-\zeta)^{\alpha-2}}{\Gamma(\alpha-1)}\hbar'(\zeta)d\zeta={}_a^C D_t^{\alpha}\big(\hbar(t)\big) \tag{8}$$

Thus, the Caputo fractional order derivative is inherently memory-dependent, and it clearly demonstrates that fractional derivatives are non-local in nature. This non-locality implies that the current value of the derivative at a given time depends not only on the instantaneous state of the function but also on its entire history, thereby capturing long-term effects and hereditary properties of the system. The fractional order α governs the memory strength of the system. When $\alpha \to 1$, the system becomes weak in the sense of memory, and when $\alpha=1$, the system becomes a completely memoryless case, equivalent to the classical integer-order derivative. Conversely, smaller values of α signify a stronger memory effect, indicating that the system's present state is more heavily influenced by its history. Different categories of memory can be characterized based on the fractional-order derivative (or integral) α, such as strong memory, weak memory, long memory, and short memory effects. These classifications are summarized in Figure 1.

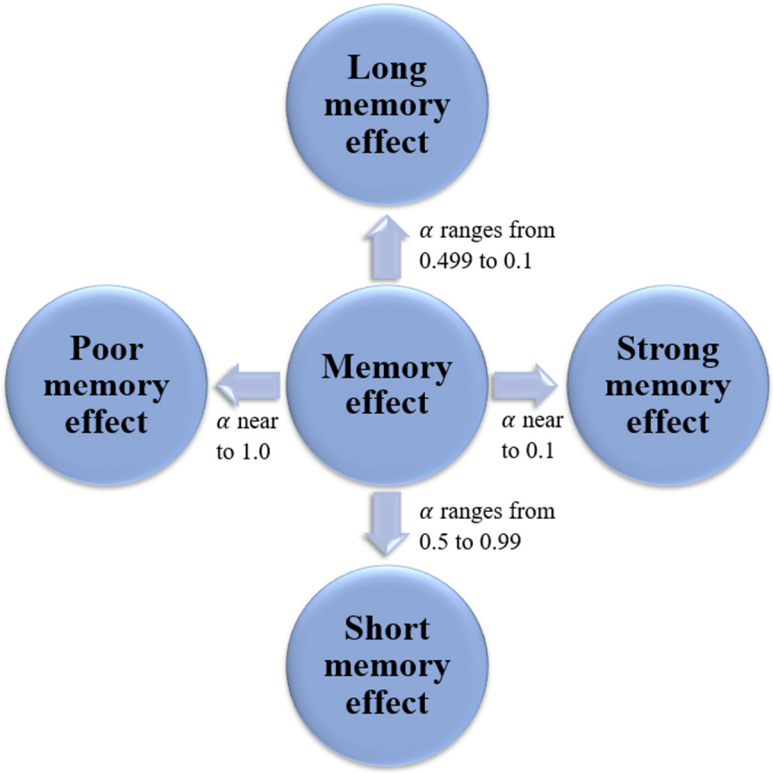


Fig. 1. Concept of memory depending on the range of memory index [25]

To illustrate the concepts of short- and long-memory effects in a real-life business context, consider the behavior of a shopkeeper. At times, a seller may offer counterfeit or duplicate products,

such as computer components or medicines, at prices significantly lower than the original branded items. Although this initially boosts sales, such products are not durable, leading to customer dissatisfaction. As a result, the shop's reputation suffers temporarily. However, if the shopkeeper maintains a positive attitude and strong customer relations, buyers may quickly forgive the incident, realizing that genuine products cannot be obtained at very low prices. In this case, the negative impact on reputation is short-lived, which reflects a short-memory effect. Conversely, if customers not only receive poor-quality goods but also encounter rude or unhelpful behavior from the seller, the bad experience leaves a lasting impression. In such situations, customers are unlikely to return, causing long-term harm to the business. This scenario represents a long memory effect within the inventory or business system.

3. Classification of Models

The term "inventory" broadly denotes the stock of goods that an organization maintains, which may include raw materials, work-in-progress items, or finished products, held to facilitate future production, sales, or distribution activities. Within the domain of operations and supply chain management, inventory serves as a critical component for maintaining balance between demand and supply. Inventory models are mathematical frameworks designed to optimize the balance between demand and supply by determining how much to order (optimal order quantity) and when to order (inventory cycle time). There exist several types of inventory models that assist organizations in systematically planning, managing, and controlling their stock levels with greater efficiency. The primary objective of these models is to minimize the costs associated with ordering, holding, and shortages, while simultaneously ensuring that customer demand is fulfilled without interruption. Among the various models developed, the EOQ model and the Economic Production Quantity (EPQ) model are regarded as the most widely applied and highly effective tools for formulating inventory control policies.

The EOQ model is a classical framework that determines the optimal order quantity that minimizes the total cost or maximizes total profit by balancing inventory-related costs. This model operates as a structured and analytical method to decide how much stock should be ordered at a time. In contrast, the EPQ model is often referred to as the production lot-size model. It provides a mechanism for identifying the optimal production lot size that minimizes total inventory-related costs by considering both production and consumption rates. This makes the EPQ model particularly relevant and beneficial in manufacturing environments where continuous production is a key feature. Together, the EOQ and EPQ models form the foundation of inventory management theory. They not only offer essential insights into cost minimization strategies but also serve as the basis for numerous advanced and modified models that incorporate real-world complexities such as uncertainty, shortages, deterioration, and fuzzy or fractional environments.

3.1 Economic Order Quantity Models

The simple EOQ model was first introduced by Harris [1] in 1913, providing a closed-form square root formula for the optimal order size. Over the past century, researchers have progressively extended this classical model by incorporating more realistic features to address practical supply chain scenarios. One of the most studied extensions is the inventory of deteriorating items, where products such as food, pharmaceuticals, and chemicals lose value or usability over time. Models have been developed to integrate deterioration in decision-making [26,27]. The consideration of shortages has enriched EOQ-type models, enabling firms to evaluate trade-offs between backordering costs and lost sales [28]. In parallel, the introduction of trade credit policies, where suppliers allow retailers

delayed payment. EOQ models with trade credit are now frequently combined with deterioration and demand variations, reflecting supply chain finance practices [29]. Firms may face limited capacity in an owned warehouse and should rely on a rented warehouse with higher holding costs. So, the two-warehouse EOQ model is relevant for wholesalers and retailers handling large volumes of deteriorating or seasonal goods [30]. In addition, suppliers are incentivized to place larger orders by reducing the per-unit cost once a threshold quantity is reached, by offering an all-units quantity discount policy. Roy *et al.* [31] formulated an EOQ model for deteriorating items with the effects of a two-level trade-credit policy. They have considered an inspection policy for imperfect production. Pervin *et al.* [32] developed an EOQ model for deteriorating items with declining demand and trade-credit policy. Integrating EOQ with all-units discounts enables firms to optimize total procurement cost while balancing holding cost increases [33].

3.2 Economic Production Quantity Models

The classical EPQ model, introduced by Taft [34], was originally developed under ideal assumptions such as perfect production and constant demand. Over the years, a wide range of research has been carried out to extend this framework to address the difficulties experienced in real-world production and supply chain systems. For instance, stock-dependent and demand-sensitive production rates in inventory control policies have been formulated to reflect contexts where production decisions are influenced by inventory availability or uncertain demand patterns. Su & Lin [35] presented a production inventory system considering the combined effect of demand rate and stock level on production rate. Their proposed model substantially enhances profitability compared to a constant production rate. Sharma *et al.* [36] analysed a production inventory model with a demand-dependent production rate for items whose life cycle is low and the degradation rate is very high. Another major research direction focuses on imperfect production systems, where issues such as defects, rework, and inspection delays are unavoidable in practice. Porteus [37] examined the effect of defective items on the classical EPQ model. In his study, the transition time between an in-control state and an out-of-control state of the production process was assumed to follow an exponential distribution, with defective units being detected and immediately replaced. Rahaman *et al.* [38] discussed an EPQ model assuming a linearly time-dependent imperfect production rate. In parallel, sustainability-oriented research has integrated remanufacturing and reverse logistics into EPQ models.

Roy *et al.* [39] developed an inventory model that incorporated the remanufacturing of returned items collected from customers, along with the recycling of imperfect products generated during the production process. Building on this, Sharma *et al.* [40] examined a reverse logistics inventory system in which both manufacturing and remanufacturing activities were influenced by the demand function. In their study, remanufactured items were assumed to be of comparatively lower quality than newly manufactured ones, and they demonstrated that remanufacturing remains profitable. Barman *et al.* [41] formulated an EPQ model considering the covid pandemic situation. Paul *et al.* [42] studied the impact of multiple prepayments and green investments on an EPQ model. More recently, Haque *et al.* [43] proposed an imperfect production-based inventory model that considered remanufacturing with price- and green-level-oriented demand. Their findings indicated that remanufacturing could serve as a cost-effective alternative to manufacturing, while simultaneously contributing to environmental sustainability. Closely related to this trend is the emergence of green EPQ models, which internalize environmental concerns such as carbon emissions reduction into production planning. Chen *et al.* [44] analytically demonstrated that carbon reduction policies can effectively lower emissions within inventory control systems, even with relatively minor adjustments

to operational strategies. Their analysis was carried out under three distinct carbon tax regimes, where they evaluated how different policy scenarios influence operational decisions, either by reducing emission-related costs or by enhancing profit margins. In a related study, Ruidas *et al.* [45] introduced an inventory model that incorporated defective production and carbon-emission parameters represented as interval-valued numbers. They further analyzed the system under four different carbon reduction policies, providing insights into the impact of environmental regulations on sustainable inventory management. Paul *et al.* [46] studied a sustainable EPQ model with remanufacturing of returned products. Collectively, these studies highlight that embedding carbon-reduction technologies and environmental policies into EPQ models not only ensures regulatory compliance but also improves operational efficiency and profitability.

3.3 Models with Different Types of Demand

In the field of inventory management, demand is considered a constant parameter to facilitate analytical tractability. However, in practical situations, demand is rarely stable and often fluctuates over time or responds directly to variations in selling price. Time-dependent demand patterns may emerge from factors such as seasonality, product life cycles, promotional campaigns, or shifting market conditions, whereas price-dependent demand arises from customer sensitivity to pricing strategies and competitive pressures. Considering these dynamic demand behaviors within inventory models enables decision-makers to capture market realities more effectively. Sana [47] experimented with an inventory model with price-dependent demand, taking fully backlogged shortages. In that model, the author discussed the influence of a stochastic selling price with a general distribution function on the demand and the overall optimal decision on inventory management. Pal *et al.* [48] studied the effect of inflation during a store shortage in an inventory lot cycle. They also addressed the delayed payment scenario on similar hypotheses about the demand function. Pal *et al.* [49] investigated a supply chain model with price-dependent demand and no shortage, where they discussed the decision-making under Stackelberg and vertical Nash structures. Shaikh *et al.* [50] allowed a fully backlogged shortage in demand in their study of the inventory model, where demand is considered a function of stock and price. That work was extended by Mashud *et al.* [51], allowing a partially backlogged shortage to replace a fully backlogged shortage. Mishra *et al.* [52] discussed the deterioration of commodities and possible preservation policies in an inventory model with demand as a function of price and stock. Khan *et al.* [53] also discussed the influence of the promotion of products on demand and inventory decisions in an inventory model for seasonal products with a fixed life span. Chen *et al.* [54] examined the inventory management of deteriorating items considering price, time, and stock-level dependent demand simultaneously. The study demonstrated that variations in the demand rate exert the most significant adverse effect on average profit per period, whereas fluctuations in the item deterioration rate were found to have the least impact.

3.4 Models with Shortages

In inventory control policy, a shortage or stockout refers to the situation where the on-hand inventory level drops to zero, and demand continues to occur, resulting in unfulfilled demand during the replenishment cycle. If every unit of unfulfilled demand during the shortage period is fulfilled later once the inventory is replenished, then this situation is considered a fully backordered or complete backlogging shortage. If some fraction of the unfulfilled demand during a shortage is backordered, while the remaining portion is considered lost due to customer impatience or substitution behavior, then this situation is considered a partial backordered shortage. Hadley &

Whitin [4] first introduced the notion of backordering into inventory control policy in the book entitled 'Analysis of Inventory Systems'. Yadav *et al.* [55] work on the memory sectors of demand and shortage dynamics with the help of optimization of fractional-order DE. Montgomery *et al.* [8] presented an inventory model with both deterministic and stochastic demand rates, considering backorders and lost sales policy. After introducing the work of Montgomery *et al.* [8], several researchers have proposed basic lot-sizing models incorporating partial backordering. Pervin *et al.* [56] formulated an integrated EOQ model with variable holding cost and shortages with partial backlogging and found the impact of a two-level trade credit policy. Next, Wang & Cheng [57] introduced an inventory system with a constant demand pattern considering partial backlogged shortages. An inventory model with a partial backordering policy was presented by Park [58]. They described the partial backordering situation as one in which a portion $\beta(0 \leq \beta \leq 1)$ of the demand is fulfilled later through backordering, while the remaining portion $1-\beta$ is considered lost. When $\beta = 1$, the model represents a fully backordered scenario, and when $\beta = 0$, it corresponds to a completely lost sales situation. Abad [59] introduced one of the earliest studies to consider the customer waiting-time-dependent backordering rate in pricing and lot-sizing inventory models. In this study, the backlogging rate $\beta(\tau)$ is defined as a decreasing function of waiting time τ, expressed as $\beta(\tau) = k_0 e^{-k_1\tau}$ or $\beta(\tau) = \frac{k_0}{1+\tau k_1}$, where $\tau, k_1 > 0$ and $0 < k_0 \leq 1$. Momena *et al.* [60] presented a two-warehouse-based inventory system of perishable items, availing a trade credit policy, all-unit quantity discount, considering a waiting-time-dependent backlogging rate. In this study, a time-dependent holding cost per unit was introduced, and it was found that storing inventory in a rented warehouse becomes preferable when deterioration in the owned warehouse increases, even if the rented warehouse is more expensive.

3.5 Limitations of Classical Models

3.5.1 Inability to capture long-memory behavior

Classical integer-order inventory models are designed by assuming that system dynamics depend only on current conditions and ignore the impact of past events. In practice, inventory systems are affected by the memory behavior of consumers or suppliers. These may lead to disruptions like shortages, lead-time increases, and demand fluctuations in the inventory control system. For example, a supply chain disruption may influence inventory replenishment time across multiple cycles, yet classical models fail to capture this persistence.

3.5.2 Oversimplified assumptions of instant replenishment

A common assumption in traditional inventory models is instantaneous replenishment, which means stock levels are restored immediately after order placement. Although this assumption fits mathematically, it is not always consistent with practical realities. Advanced supply chain networks are influenced by various factors like supplier-side uncertainties, imperfect production, deterioration rate of the items, and customs clearance, which lead to non-instantaneous replenishment. These types of models may suggest replenishment policies that are infeasible in practice, which may increase the risk of stockouts, excessive holding costs, or inefficiencies under delayed replenishment conditions.

3.5.3 Lack of realism in representing real-life inventory patterns

Traditional inventory models are formulated under assumptions such as constant demand, fixed holding costs, perfect production rate, no deterioration, and deterministic supply conditions. However, real-world inventory systems are subject to more complex dynamics, including demand

variability by selling price, stock level, green level, warranty time, advertisement frequency, time-dependent holding costs, supplier disruptions, product deterioration, and time-dependent imperfect production rate. The exclusion of these factors reduces the predictive accuracy and practical relevance of classical models in supply chain environments. Although the classical models serve as valuable theoretical foundations, their lack of realism decreases the application of the EOQ and EPQ models in terms of uncertain and complex environments.

4. Comparative Analysis

A large body of literature has utilized the concept of fractional calculus in the formulation of inventory control models. Also, inventory systems often involve inherent uncertainty in their parameters. To address this, several researchers have incorporated fuzzy Logic alongside fractional calculus in inventory modelling. In this section, a comparative summary is presented of existing inventory models developed using both fractional calculus and the fuzzy fractional calculus approach.

4.1 Fractional Inventory Models

The fractional order EOQ or EPQ inventory models extend classical inventory theory by introducing fractional calculus into the governing equations, enabling the modelling of memory effects and history-dependent dynamics in replenishment and production systems. Traditional EOQ and EPQ models assume that changes in inventory levels depend only on the current state and parameters such as demand rate, holding cost, and ordering or setup cost. However, in many real-life systems, such as manufacturing processes with setup delays, agricultural produce with storage memory effects, or industrial parts with gradual wear, inventory dynamics depend on the entire history of stock levels and operational conditions. The fractional derivative framework captures this effect by replacing the standard first-order derivative with a fractional derivative of order $\alpha \in (0, 1]$, where α measures the degree of memory in the system. The incorporation of fractional calculus into inventory modelling was notably advanced by Das & Roy [61], who applied both the Riemann–Liouville and Caputo definitions of fractional derivatives to reformulate the EOQ model and solved the Fractional Differential Equation (FDE) using the Laplace transform method. Their generalized framework retained the traditional EOQ model as a special case when the fractional order was unity, thereby providing a broader analytical structure for inventory systems with constant demand and cost parameters. Building on this foundation, Das & Roy [62] extended the model by replacing the constant demand assumption with a linearly time-dependent demand rate, enabling its application to environments where demand evolves. Subsequently, Das & Roy [63] sought to construct an EPQ model within the fractional calculus framework; however, this attempt did not yield a corresponding optimization technique for executing numerical simulations. In the years that followed, Pakhira and collaborators substantially expanded this line of research by examining EOQ models under fractional calculus, where the memory effect of inventory systems was explicitly considered. Their works, including Pakhira *et al.* [64], Pakhira *et al.* [65], and Pakhira *et al.* [66], focused on linearly time-dependent demand, while Pakhira *et al.* [67] introduced a quadratic time-dependent demand pattern to investigate how system memory influences optimal replenishment policies. Moreover, deterioration effects, representing the decline in item quality over the inventory cycle, were incorporated into fractional EOQ models by Pakhira *et al.* [68] and Pakhira *et al.* [69], thereby enhancing the descriptive power and practical relevance of such models in real-world inventory management contexts. Rahaman *et al.* [70] proposed a generalized EPQ model that incorporates deterioration, employing the Mittag–Leffler function within a real distinct pole approximation framework for both analytical and numerical optimization. Extending this line of research, Rahaman

et al. [71] formulated a memory-oriented EPQ model based on Caputo's fractional derivative, where demand is influenced simultaneously by price and stock level, while the production rate is assumed to be stock-dependent. To address the computational complexity of the model, they applied a modified artificial bee colony algorithm for numerical simulation, rather than relying solely on traditional analytical methods. Jana & Das [72] developed a memory-dependent inventory model that considers non-instantaneous deterioration of items, where the deterioration process begins after a certain shelf-life period. The model further incorporates stock-dependent demand, reflecting the realistic scenario in which consumer demand increases with higher inventory levels, and allows for partially backlogged shortages, capturing situations in which a fraction of unsatisfied demand is postponed rather than completely lost. This framework highlights the combined effect of memory, deterioration dynamics, and demand–shortage interactions on inventory decision-making. Rahaman *et al.* [73] developed a lot-sizing model in which demand is influenced by inventory level and price, incorporating memory effects and allowing for completely backlogged shortages. They examined four distinct model variations based on different combinations of memory and shortage assumptions, and their analysis revealed that the presence of system memory tends to reduce overall profitability. Santra *et al.* [74] presented an EOQ model of deteriorating items with promotional effort and reliability-dependent demand using the Caputo-Fabrizio fractional derivative approach. Jana *et al.* [75] introduced a memory-based inventory model for deteriorating items considering backlogged shortages. A comparative summary of the fractional inventory model is presented in Table 1. Their findings revealed that memory significantly influences the total average cost, with long-memory effects yielding more substantial results compared to short-memory or memoryless systems.

Table 1
Comparative summary of fractional inventory models

Authors	Publication year	Model type	Fractional approach	Demand type	Deterioration	Shortage	Optimization method
Das & Roy [61]	2014	EOQ	Caputo fractional derivative and R-L fractional integration	Constant	×	×	Geometric programming
Das & Roy [62]	2015	EOQ	Caputo fractional derivative and R-L fractional integration	Linear time-dependent as $D = at + b$	×	×	Geometric programming
Das & Roy [63]	2017	EPQ	Caputo fractional derivative and R-L fractional integration	Constant	×	×	Geometric programming
Pakhira *et al.* [64]	2018	EOQ	Caputo fractional derivative	Polynomial of t^{α} as $D = a + bt^{\alpha}$	×	×	Primal geometric programming
Pakhira *et al.* [67]	2018	EOQ	Caputo fractional derivative	Polynomial of t^{α} as $D = a + bt + ct^{2}$	×	×	Primal geometric programming

Table 1 (*continued*)

Authors	Pub. year	Model type	Fractional approach	Demand type	Deterioration	Shortage	Optimization method
Pakhira *et al.* [68]	2020	EOQ	Caputo and R-L fractional derivative	Time-dependent as $D = aE_\alpha(-\lambda t^\alpha)$, λ is constant and E_α is the Mittag-Leffler function)	Constant	Backlogging rate is waiting time dependent as $S = \frac{1}{(1+(T-t)^\alpha \rho)}$, ($\rho$ is backlogging parameter)	Primal geometric programming
Rahaman *et al.* [70]	2020	EPQ	Caputo and R-L fractional derivative	Constant	Constant	×	Geometric programming
Rahaman *et al.* [71]	2020	EPQ	Caputo fractional derivative	Price and stock dependent demand as $D = a - bp + cI(t)$, (p is price and $I(t)$ is stock level)	constant	×	Artificial bee colony (ABC) algorithms
Jana & Das [72]	2021	EOQ	Caputo and R-L fractional derivative	Stock-dependent demand as $= a + bI(t)$	Constant	Backlogging rate is waiting time dependent as $S = E_\alpha(-\delta(T-t)^\alpha)$, ($\delta$ is a positive constant)	Non-linear programming using Matlab Software
Rahaman *et al.* [73]	2022	EOQ	Caputo fractional derivative and R-L fractional integration	Price- and stock-dependent demand as $D = a - bp + cI(t)$	×	Completely backlogged	Non-linear programming using Mathematica Software
Santra *et al.* [74]	2023	EOQ	Caputo–Fabrizio fractional derivatives.	Promotional effort and reliability-dependent demand as $D = a(1 + \rho r)$, (ρ is the coefficient of promotional efforts and r is the coefficient of reliability	Constant	×	Non-linear programming using Matlab Software
Jana *et al.* [75]	2024	EOQ	Caputo fractional derivative	Constant	Constant	Completely backlogged	Non-linear programming using Matlab Software

*Parameters a, b, and c in the demand type are positive constants; '×' means not considered

4.2 Fuzzy Fractional Inventory Models

There is always some degree of uncertainty in observations, measurements, and decision-making processes. To approach real-world processes, computational simulations are often used. Dealing with such vague scenarios requires mathematical tools. Fuzzy logic could potentially fulfill this need. Bellman & Zadeh [21] integrated the concept of fuzziness into decision-making issues. Park [22] was the first to introduce a fuzzy set-theoretic understanding of the EOQ model. In the inventory control

system, the demand pattern of a particular commodity is uncertain. Uncertainty is present in every decision-making scenario. If this uncertainty is considered in the fractional calculus, the environment becomes fuzzy fractional. Agarwal *et al.* [76] pioneered the application of Riemann-Liouville differentiability in an uncertain setting to demonstrate fractional differential equations (FDEs). This innovative work opened a new direction in research focusing on studying FDEs within uncertain environments. To the best of our knowledge, Pakhira et al. [23] first proposed a fractional derivative-based inventory model incorporating memory effects with linearly time-dependent demand under a fuzzy environment. To handle the fuzziness in the parameters, they employed two widely used defuzzification techniques, namely the Signed Distance (SD) method and the Graded Mean Integration (GMI) method. This study opened a novel research direction by integrating fractional calculus and fuzzy logic within a single inventory framework, thereby addressing both memory-dependent characteristics and uncertainty in decision-making simultaneously. Rahaman *et al.* [24] interpreted a memory-based EPQ model for deteriorating items under a fuzzy environment. They used the Riemann–Liouville fractional derivative to discuss the model and proposed a new defuzzification formula to compare the results. Rahaman *et al.* [77] presented a memory and learning-based EOQ model with price-dependent demand using a triangular dense fuzzy lock set. They also presented the sense of generalization through different aspects of fuzziness and fractional calculus. Rahaman *et al.* [38] discussed the solution strategy of fuzzy fractional-order linear homogeneous differential equations using Caputo H-differentiability and analysed an EOQ model with stock-dependent demand using this theory. Next, several authors' studies on the EOQ model with non-linear time-dependent demand under a fully backordered shortage policy using fractional calculus and a triangular dense fuzzy environment. Rahaman *et al.* [77] introduced a generalized EOQ model incorporating fuzzy uncertainty and fractional differentiability based on the Caputo gH derivative and the RL integration, and discussed it using a fuzzy fractional differential equation (FuFDE) approach. Pakhira *et al.* [78] formulated an inventory model within the described environment, explicitly accounting for carbon emissions arising from ordering, holding, and shortage-related activities. Their analysis revealed that profit levels are generally higher when the system exhibits strong memory effects, whereas under poor memory conditions, the profitability tends to decline. Momena *et al.* [25] discussed a memory-dependent inventory system under a fuzzy environment, considering price-dependent demand and complete backlogging shortages. They considered the selling price as a triangular fuzzy number, and defuzzification is performed through the SD and GMI methods. The findings revealed that under the SD method combined with the memoryless case, businesses require a longer operational period to reach the minimum total average cost. In contrast, when applying the GMI method in the memoryless case, the minimum total cost is achieved in a comparatively shorter duration. Haque *et al.* [79] developed an EOQ model with price- and green level-dependent demand using FDE under a neutrosophic fuzzy environment. Recently, Haque *et al.* [80] presented another inventory model with green level- and stock level-dependent demand. A comparative summary of the fuzzy fractional inventory model is presented in Table 2.

5. Methodologies

5.1 Analytical Techniques

5.1.1 Laplace transform methods

Laplace transform methods are powerful mathematical techniques used in engineering, physics, mathematics, and economics to simplify the analysis of fractional-order differential equations. Bázsa *et al.* [84] formulated an inventory model and solved the model with the help of the Laplace

transform. Das *et al.* [85] developed an interval Laplace transform and its application in a production inventory model.

Table 2
Comparative summary of fuzzy fractional inventory models

Authors	Pub. year	Model type	Uncertainty type	Demand type	Deterioration	Shortage	Solution method
Pakhira *et al.* [23]	2019	EOQ	Triangular fuzzy	Linear time-dependent as $D = a + bt$	×	×	Fuzzy arithmetic operation
Rahaman *et al.* [24]	2020	EPQ	Fuzzy	Constant	Constant	×	Fuzzy fractional differential equation
Rahaman *et al.* [81]	2021	EOQ	Triangular dense fuzzy lock set	Price depended as $D = a - bp$	×	×	Fuzzy fractional differential equation
Rahaman *et al.* [82]	2021	EOQ	Triangular fuzzy	Stock dependent as $D = a\, I(t)$	×	×	Fuzzy fractional differential equation
Rahaman *et al.* [83]	2021	EOQ	Triangular dense fuzzy	Demand is uniform on the inventory running phase and gradually decreases during the shortage period	×	Completely backlogged	Fuzzy fractional differential equation
Rahaman *et al.* [77]	2022	EOQ	Trapezoidal fuzzy	Constant	×	×	Fuzzy fractional differential equation
Pakhira *et al.* [78]	2024	EOQ	Triangular fuzzy	Constant	×	Completely backlogged	Fuzzy arithmetic operation
Momena *et al.* [25]	2025	EOQ	Triangular fuzzy	Price-dependent as $D = ap^b$	×	Completely backlogged	Fuzzy fractional differential equation
Haque *et al.* [79]	2025	EOQ	Trapezoidal neutrosophic fuzzy	Price and green level dependent as $D = a - bp + cg$ (g is green level)	×	×	Neutrosophic Fractional differential equation approach
Haque *et al.* [80]	2025	EOQ	Triangular neutrosophic	Stock level and green level dependent as $D = a + bI(t) + cg$	Constant	×	Neutrosophic Fractional differential equation approach

5.1.2 Adomian decomposition method

The Adomian decomposition method serves as a powerful tool in analyzing fractional-order inventory problems, offering both analytical insight and computational efficiency for complex supply chain models involving memory, uncertainty, and nonlinearity. The Adomian Decomposition Method

provides rapidly convergent solutions for fractional differential equations. Moreover, it is easy to implement computationally and analytically. It also offers flexibility to incorporate deterioration, trade credit, shortages, and carbon emissions under fractional-order dynamics. It captures memory effects in supply chain systems, which are crucial in modelling real-world inventory processes. In the literature, Susanto *et al.* [86] developed a fractional model using the Adomian Decomposition Method. Mahmood *et al.* [87] formulated a multi-dimensional time fractional model using the Laplace Adomian Decomposition Method. González-Gaxiola *et al.* [88] found the pricing condition in a fractional model using the Adomian Decomposition Method.

5.1.3 Homotopy perturbation method

The Homotopy Perturbation Method is a powerful analytical tool for solving fractional-order inventory problems, offering a balance between simplicity, accuracy, and efficiency, while effectively modelling memory-driven behavior in modern supply chains. Consider a fractional-order deteriorating inventory model where demand is time-dependent, and deterioration follows a fractional decay law. Using the Homotopy Perturbation Method, the governing fractional differential equation is decomposed into a homotopy structure, and the solution is expressed as a series in powers of the embedding parameter. The first few terms yield an excellent approximation to the optimal inventory trajectory, minimizing cost under memory-dependent dynamics. Jeyanthi *et al.* [89] developed an inventory model with price-dependent demand and an exponential deterioration rate. The model is enhanced to a fractional derivative using the Caputo Derivative method and solved using the Homotopy Perturbation Method. Viswanath [90] analysed some inventory models by using the Homotopy Perturbation Method.

5.2 Numerical Optimization

5.2.1 Genetic algorithms

The genetic algorithm is a powerful optimization tool for solving complex inventory models that cannot be addressed analytically. Its ability to handle nonlinearity, uncertainty, and multiple objectives makes it particularly valuable in modern supply chain management and sustainable inventory systems. Gupta *et al.* [91] studied an application of the Genetic Algorithm in solving an inventory model with advance payment and interval-valued inventory costs. Iwasokun & Alimi [92] developed a genetic algorithm model for inventory control and management that addresses some of the limitations. Aashish & Kumar [93] derived a deteriorating inventory model with price-dependent demand and time-dependent holding cost in a fuzzy environment. A genetic algorithm was used to find the optimal strategies of the inventory model, such as minimizing the total cost and optimal order quantity.

5.2.2 Particle swarm optimization

Particle Swarm Optimization (PSO) is a population-based metaheuristic optimization technique inspired by the social behavior of birds flocking or fish schooling. It is widely applied to solve complex and nonlinear inventory models where analytical or exact methods are difficult to apply. Its balance of simplicity, speed, and accuracy makes it an attractive alternative to traditional optimization and other evolutionary algorithms in supply chain and inventory management research. Tsai & Yeh [94] presented a PSO approach for inventory classification problems where inventory items are classified based on a specific objective, such as minimizing costs, maximizing inventory turnover ratios, and maximizing inventory correlation. Chan *et al.* [95] proposed an inventory model that can enhance food quality by making the best possible food quality accessible to customers by constructing a

sustainable logistics system, considering its environmental impact and ensuring the customer demand is fulfilled as fast as possible.

5.2.3 Lagrangian and dynamic programming approaches

The Lagrangian and Dynamic Programming approaches remain foundational in inventory optimization. While the Lagrangian method simplifies constrained optimization, dynamic programming provides powerful recursive solutions for multi-stage and stochastic problems. Together, they offer robust analytical tools for solving complex inventory and supply chain models.

5.3 Simulation-Based Techniques

5.3.1 Monte Carlo simulations for uncertain demand

Monte Carlo Simulation (MCS) is a computational technique that uses random sampling and probability distributions to model and analyze systems under uncertainty. In inventory management, it is widely applied when demand, lead time, deterioration, or other parameters are uncertain and cannot be captured accurately by deterministic models. It provides insights into risk, variability, and performance under stochastic environments, making it invaluable for robust supply chain decision-making. Cruz *et al.* [96] developed a mathematical model and a Monte Carlo Simulation routine that aims to assist managers in establishing a production and inventory management plan for a perishable product, considering stochastic and non-stationary demand. Elsayed *et al.* [97] used a Monte-Carlo simulation approach to model the blood supply chain under both supply and demand uncertainty. Two different blood supply strategies were generated to analyze the blood supply chain performance.

5.3.2 Agent-based modelling for decentralized systems

Agent-Based Modelling (ABM) is a computational simulation approach where a system is modelled as a collection of autonomous decision-making entities, called agents. Each agent operates according to defined behavioral rules, interacts with other agents, and adapts to changing conditions. In decentralized inventory and supply chain systems, ABM is especially valuable for analyzing emergent behavior, coordination, and adaptive decision-making. Agent-Based Modelling provides a powerful framework for analyzing decentralized inventory systems, where individual actors make independent decisions under uncertainty. By simulating agent interactions, ABM reveals how local strategies influence overall supply chain efficiency, adaptability, and resilience. Kessentini *et al.* [98] designed an Agent-Based modelling and simulation of inventory disruption management in the supply chain. Helo & Rouzafzoon [99] formulated a model to minimize transportation costs in collecting tree logs from several regions and delivering them to the nearest collection point.

6. Applications of Fractional Inventory Models

This section presents the applicability of the fractional-order inventory models in various real-world scenarios. It outlines recent research trends, key advantages of fractional-order models, and potential domains where such approaches can be implemented.

6.1 Recent Research Trends in Applications

6.1.1 Integration with carbon emission constraints

A recent research trend in inventory control is the integration of carbon emission constraints into integer-order inventory models as well as fractional-order inventory models. Enabling carbon emission restrictions in fractional calculus-based inventory models provides a more realistic

representation of system dynamics under sustainability concerns. In this context, researchers have examined the integration of carbon emissions, carbon emission policies, and emission reduction technologies into inventory control strategy by Mishra *et al.* [100] and Ali *et al.* [101]. This integration represents a sustainable and memory-sensitive inventory control strategy aligning with global environmental objectives.

6.1.2 Incorporation of green supply chain management

Another recent research direction in inventory modelling is the integration of supply chain management with green practices such as environmental sourcing, eco-friendly packaging, energy-friendly production, adoption of renewable resources, remanufacturing, and reverse logistics by Paul *et al.* [102] and Sepehri & Gholamian [103]. Fractional-order dynamics also provide a powerful mechanism for capturing the influence of past production, demand patterns, and storage activities on both inventory performance and environmental outcomes, allowing a more realistic assessment of long-term effects. Recent studies have increasingly combined fractional calculus with green supply chain strategies to examine the trade-off among profitability, sustainability and customer satisfaction [79]. This growing field of research gives valuable tools for decision-makers to design environmentally responsible supply chain systems.

6.1.3 Use of cloudy fuzzy numbers for greater uncertainty modelling

A recent advancement for the inventory control problem in an uncertain environment is the introduction of cloudy fuzzy or dense fuzzy logic [104,105]. Human intelligence involves learning from repeated tasks, where habitual practice improves performance and enhances decision-making quality. The cloudy fuzzy number extended the traditional fuzzy numbers by capturing the human learning experience through the precision of fuzziness [106,107]. Classical fuzzy approaches are effective in handling imprecise parameters related to demand rate, lead time, or inventory costs [108]. Their incorporation into inventory models enhances the robustness and adaptability of decision-making under uncertain market environments.

6.1.4 Combining fractional models with stochastic demand and inflation effects

Another emerging trend of research is developing inventory control considering stochastic demand and inflationary effects [109]. Stochastic demand means randomness in customer demand rates, and inflation implies changes in prices of inventories over time. Both have a significant influence on inventory-related decisions. By applying these factors, researchers have developed a fractional-order-based inventory problem that balances profitability, service levels, and long-term stability.

6.2 Advantages of Fractional-Order Models in Applications

6.2.1 Better fit for real-world nonlinear, memory-driven processes

Classical inventory models were developed considering linear and memoryless assumptions. In practice, it does not fully reflect real-world inventory problems. Because, most of the supply chain scenarios are influenced by memory and past experiences in customer demand, unexpected shortages, or replenishment actions. Fractional-order inventory control models are well-suited to capture these memory-dependent effects, where the fractional order governs the memory sensitivity of the system. Fractional derivatives can incorporate experience into the current decision-making scenario. It accurately represents how previous memory affects present inventory levels and

enhances the precision and reliability of the fractional-order inventory problem compared to the classical integer-order problem [110,111].

6.2.2 Increased flexibility in modelling long-term behaviour

Fractional-order inventory models can also represent the long-term behaviour of the system with greater flexibility. Classical integer-order models usually concentrate on short-term cycles and may fail to capture long-term influences, such as gradual demand patterns or inflationary impacts on inventory-related costs and shortages. But fractional models allow arbitrary order differentiation of the on-hand inventory level function. These characteristics permit precise adjustment of the system's memory behaviour, which helps more realistic modelling of the inventory in long-term trends. Fractional order inventory models are important in organisations where inventory planning extends long cycle horizons and strategic decisions must consider both short-term fluctuations and long-term patterns.

6.2.3 More realistic representation of deterioration and restocking delays

Classical inventory systems deal with items, most of which deteriorate over time, such as fruits and vegetables, pharmaceuticals, or chemicals. It also experiences delays in replenishment due to uncertain lead times. Traditional inventory models are designed to simplify these dynamics, which can result in sub-optimal inventory policies. In contrast, fractional-order models capture the gradual decay of items and the effects of delayed replenishment. By incorporating memory and long-term dependencies, these models can represent the impact of deterioration and replenishment gaps across multiple inventory cycles. This makes them well-suited for perishable goods and for industries with significant logistics delays, ultimately enabling improved stock allocation and reduced waste.

6.3 Potential Application Areas

i. The review consolidates existing fractional inventory models, providing researchers with a structured overview of developments, methodologies, and solution techniques. Besides, the comparative discussion of methods like Adomian Decomposition, Homotopy Perturbation, Genetic Algorithms, and Monte Carlo simulations helps researchers choose suitable approaches for solving complex fractional inventory problems.
ii. The review can be used as a reference in graduate-level courses on operations research, supply chain management, and applied mathematics.
iii. The insights from fractional inventory models help managers address real-world issues like uncertain demand, deteriorating items, trade-credit policies, and carbon emission constraints.
iv. The paper guides models incorporating environmental impacts, which can be applied in industries seeking to align with green supply chain management and carbon reduction policies.
v. Fractional models reviewed in the paper are applicable in sectors like pharmaceuticals, food, and agriculture, where product deterioration and uncertainty play a critical role.
vi. Applications in electronics and spare parts management benefit from fractional modelling approaches due to their capacity to handle imperfect quality items and fluctuating demand.
vii. Policymakers can use the review's findings to promote sustainable inventory strategies by balancing economic efficiency with environmental concerns.

7. Challenges and Limitations

Although fractional inventory modelling has several important advantages, as discussed in the previous section, it also faces some challenges and limitations, which are addressed in this section.

7.1 Lack of Standardization in Solution Techniques

Fractional-order inventory models are based on fractional calculus. However, unlike classical integer-order differential equations, there is no standard method for solving fractional differential equations (FDEs). Various definitions of fractional derivatives, such as Riemann–Liouville, Caputo, Grünwald–Letnikov, and Atangana–Baleanu, produce different analytical and numerical outcomes. This creates challenges in comparing findings across studies and reduces the progress of developing generalized solution techniques. Researchers use problem-specific approaches, which reduces the broader applicability of fractional order models within different areas of inventory management.

7.2 Computational Complexity in Solving Fractional Differential Equations

Solving FDEs is more challenging than working with integer-order differential equations, as fractional derivatives are non-local operators that capture the system's historical behaviour. Each computation state depends on all prior states, which increases computational burden. Although advanced numerical techniques, such as spectral methods and finite difference schemes, these are not appropriate in terms of accuracy and efficiency. In a complex supply chain system with multiple items, deterioration effects, or uncertain demand, these computational challenges are reducing the practical implementation of fractional inventory models.

7.3 Difficulty in Parameter Estimation (Fractional Order)

Another challenge in fractional modelling is estimating the fractional order parameter, which determines the extent of memory involved in the system. Parameters in fractional order modelling do not always have a straightforward physical interpretation like classical integer order modelling; instead, these parameters are usually derived through calibration with historical data. Due to various external influences like seasonal fluctuations or demand disruptions, real-world inventory parameters are often incomplete or ambiguous. These complications make parameter estimation difficult and susceptible to overfitting. As a result, without robust parameter estimation methods, the practical reliability of fractional inventory models may be undermined.

7.4 Limited Empirical Validation in Industry

Although fractional inventory models have strong theoretical benefits, their application in the real-world industry is still limited. Most of the published research is conceptual and is validated with hypothetical datasets. There is a gap in large-scale empirical studies that confirm their effectiveness in real-world supply chains. For practical implementation, industries require models that are easily interpretable and well-suited to standard decision-making systems. In fractional inventory modelling, these criteria are not fully met. This gap between academic progress and industrial validation reduces the progress of fractional inventory models from theory into practice.

8. Future Directions

Despite the significant advancements in fractional-order inventory modelling, several research gaps remain for future investigation. The following directions can guide both researchers and practitioners toward advancing the domain.

8.1 Extension to More Complex Inventory Systems

The development of more realistic and comprehensive inventory models by incorporating trade-credit policies, quantity discount schemes, preservation technologies, carbon emission considerations, green technology adoption, and waste management strategies can be a good area of research. Furthermore, such complex models can be analysed under advanced uncertainty representation frameworks, including intuitionistic fuzzy and neutrosophic environments, to better capture imprecision and indeterminacy inherent in real-world data.

8.2 Need for Real-World Validation

One of the major limitations in existing literature is the lack of empirical validation using real industrial data. Most fractional inventory models have discussed numerical simulation using hypothetical data, which may not effectively capture the complexities of real supply chains. Future research should focus on developing case studies and industry collaborations to validate these models using real-world datasets.

8.3 Development of Hybrid Fractional-Stochastic Models

The existing literature focuses on fractional calculus and uncertainty. However, real-world inventory systems are inherently stochastic due to random demand, lead time variability, and supply disruptions. Future research should explore hybrid frameworks that combine fractional-order dynamics with stochastic processes. The development of fractional-stochastic inventory models can provide a more realistic representation of uncertainty and system memory.

8.4 Advanced Techniques for Parameter Estimation

Estimation of the values of several inventory parameters, as well as the order of the fractional derivative, remains a challenging issue due to its abstract interpretation and dependence on historical data. Future studies should focus on designing the estimation techniques, maybe by integrating machine learning, optimization algorithms, or Bayesian inference methods.

9. Conclusion

This paper provides a comprehensive review of fractional inventory problems, summarizing the state-of-the-art in models, methodologies, applications, and research challenges. By analyzing a wide spectrum of studies, it becomes evident that fractional calculus has significantly enriched inventory modelling, particularly in handling memory effects, product deterioration, uncertain demand, and sustainability considerations.

The review highlights that fractional approaches outperform classical inventory models by offering greater flexibility in capturing real-world complexities such as imperfect items, trade-credit policies, and environmental constraints. Furthermore, the integration of fractional models with optimization techniques, including decomposition methods, perturbation methods, metaheuristics, and simulation-based approaches, demonstrates their versatility in addressing both theoretical and practical problems.

Applications across diverse domains, such as healthcare, food supply chains, manufacturing, and green supply chains, showcase the growing relevance of fractional inventory models in tackling modern challenges like perishability, uncertainty, and carbon emissions. The insights drawn from this review not only consolidate existing knowledge but also provide actionable directions for future research.

In conclusion, fractional inventory modelling is an emerging and promising field that bridges mathematical theory with real-world supply chain and inventory practices. By advancing solution methods, expanding into multi-echelon and sustainable systems, and integrating with digital technologies, fractional models hold the potential to reshape the way inventory management problems are analyzed and solved. This review serves as both a roadmap and a foundation for researchers, practitioners, and policymakers aiming to develop efficient, resilient, and sustainable inventory systems in the years ahead.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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